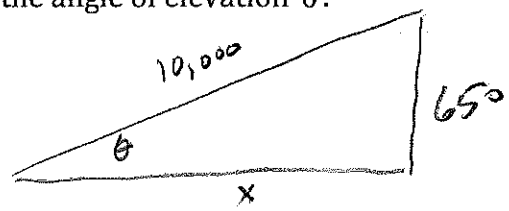
Precalculus CP 1 - Midyear Review Questions for Chapters 1, 4, 5, 6

The following are GREAT problems to do when studying for your final exam. We will have some time in class for you to work on them, but you should also spend some time on your own to be sure you understand each one. If you need more practice, try redoing your ICEs and old tests and quizzes.

1) A straight road is going slightly uphill. The road is 10,000 ft. long, and rises a total of 650 ft.

   a) Draw a picture representing the situation using a right triangle. Label the road length and the rise of theroad. Call the unknown horizontal distance x.
   Call the angle of elevation θ.



   b) Use the Pythagorean Theorem to find the horizontal distance.
   Give your answer in feet rounded to 2 decimal places.

$$x = \sqrt{(10{,}000)^2 - 650^2} = 9978.85 \text{ ft.}$$

   c) Find the angle of elevation. Use degrees and round to 2 decimal places.

$$\theta = \sin^{-1}\left(\frac{650}{10000}\right) \approx 3.73^\circ$$

   d) If a car goes up the entire hill in 6 minutes, what is the speed of the car in miles per hour?
   (there are 5280 ft. in 1 mile) (round to 2 decimal places)

$$\frac{10000 \text{ ft}}{1 \text{ min}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 18.94 \frac{\text{miles}}{\text{hr}}$$

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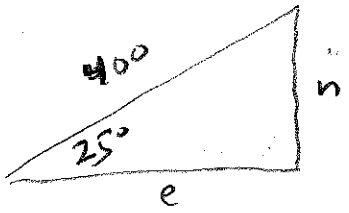
2) From take-off, an airplane travels 400 miles at an angle of 25° north of east. Find the distance that the plane traveled a) north and b) east.



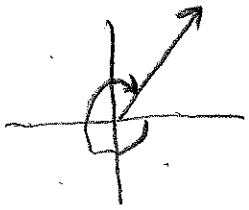
$\sin 25° = \frac{n}{400}$ $n = 400 \sin 25$

$\approx 169.05\text{ m}$

$e = 400 \cos 25 \approx 362.52\text{ m}$

For #3-4, sketch the angle in standard position. Find the degree measure equivalent. Then find two coterminal angles- one positive and one negative.

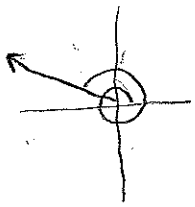
3) $\frac{-5\pi}{3} \cdot \frac{180}{\pi} = -300°$



$\frac{\pi}{3}, \frac{-11\pi}{3}$

$60°, -660°$

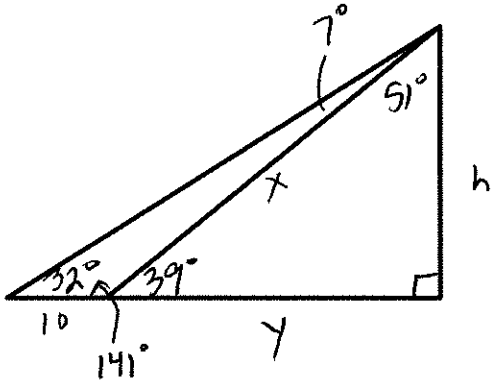
4) $\frac{17\pi}{6} \cdot \frac{180}{\pi} = 510°$



$\frac{5\pi}{6}, -\frac{7\pi}{6}$

$150°, -210°$

5) From a certain distance, the angle of elevation to the top of a building is 39°. At a point 10 meters farther from the building, the angle of elevation is 32°. Approximate the height of the building to three decimals, and find the original distance you were from the building before you moved farther away.



$\frac{x}{\sin 32°} = \frac{10}{\sin 7°}$

$x \approx 43.482257$

$\frac{h}{\sin 39°} = \frac{}{\sin 90°}$

$h \approx 27.364\text{ m}$

$\frac{y}{\sin 51°} = \frac{}{\sin 98°}$

$y \approx 33.729\text{ m}$

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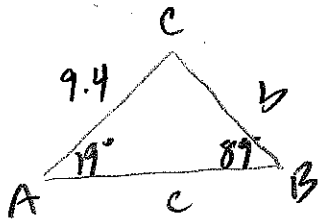
6) For the following triangles, use the law of sines and cosines to solve for all the missing sides and angles. Then find the area of each triangle. Be sure to check to see if you have two triangles in certain cases!

a) $\angle A = 19°, \angle B = 89°, b = 9.4$



$m\angle C = 180 - 19 - 89 = \boxed{72°}$

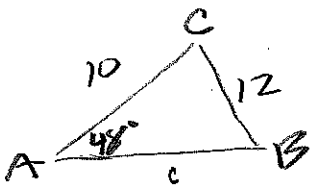
$\frac{a}{\sin 19°} = \frac{9.4}{\sin 89°} \Rightarrow \boxed{a \approx 3.06}$

$\frac{c}{\sin 72°} = \frac{9.4}{\sin 89°} \Rightarrow \boxed{c \approx 8.94}$

$A = \frac{1}{2}(9.4)(8.94)\sin 19° \approx \boxed{13.68\ u^2}$

b) $\angle A = 48°, a = 12, b = 10$

SSA!



$\frac{\sin B}{10} = \frac{\sin 48°}{12}$

$\angle B \approx 38.26°$

$\angle C \approx 93.74°$

$c = 16.11$

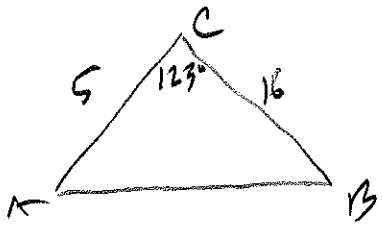
OR ~~171.74°~~ one soln!

$\frac{c}{\sin 93.74°} = \frac{12}{\sin 48°}$

$A = \frac{1}{2}(10)(12)\sin 93.74°$

$\approx 59.87\ u^2$

c) $\angle C = 123°, a = 16, b = 5$



$c^2 = 5^2 + 16^2 - 2(5)(16)\cos 123°$

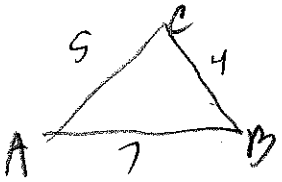
$c \approx 19.19$

$\angle A = 44.38°$

$\angle B = 12.62°$

$A = \frac{1}{2}(5)(16)\sin 123° \approx \boxed{33.55\ u^2}$

6d) $a=4, b=5, c=7$



$\angle A = \cos^{-1}\left(\frac{4^2-5^2-7^2}{-2(5)(7)}\right) = \cos^{-1}\left(\frac{29}{35}\right) \approx 34.05°$

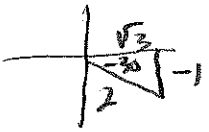
$\angle C = \cos^{-1}\left(\frac{7^2-5^2-4^2}{-2(5)(4)}\right) = \cos^{-1}\left(-\frac{1}{5}\right) \approx 101.54°$

$\angle B = 180 - \angle A - \angle C \approx 44.41°$

$A = \sqrt{8(8-4)(8-5)(8-7)} \approx 9.80\ u^2$

Use ANY OF THE FORMULAS (sum, difference, half, double) for the following questions:

7) Find the **EXACT value** of the expression – this means no decimals!



a) $\sin(-75°) = \sin(-30 + -45)$

$= \sin(-30)\cos(-45) + \cos(-30)\sin(-45)$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-\sqrt{2}-\sqrt{6}}{4}$

b) $\tan 22.5° = \tan\frac{45}{2} = \frac{1-\cos 45°}{\sin 45°} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{1}$

$= \sqrt{2}-1$

8) Write the expression as the sine, cosine, or tangent of the angle; you do not have to find the value:

a) $\sin 60°\cos 45° - \cos 60°\sin 45°$

$\sin(60-45) = \sin 15°$

b) $\dfrac{\tan 25° + \tan 10°}{1 - \tan 25°\tan 10°}$

$\tan(25+10) = \tan 35°$

c) $12 - 24\sin^2 x$

$12(1 - 2\sin^2 x)$

$= 12(\cos 2x)$

d) $\sqrt{\dfrac{1-\cos 6x}{2}} = \sin\frac{6x}{2} = \sin 3x$

9) Find the **exact value** of the trig function given that

$\sin u = \frac{3}{5}$ and $\cos v = \frac{-7}{25}$, where both u and v are in the same quadrant.

$\sin(u-v) =$

$= \sin u \cos v - \cos u \sin v$

$= \frac{3}{5}\left(\frac{-7}{25}\right) - \left(\frac{-4}{5}\right)\left(\frac{24}{25}\right)$

$= \frac{-21}{125} + \frac{96}{125} = \frac{75}{125} = \boxed{\frac{3}{5}}$

$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

$= \frac{-\frac{3}{4} + -\frac{24}{7}}{1 - \left(-\frac{3}{4}\right)\left(-\frac{24}{7}\right)}$

$= \frac{\frac{-21-96}{28}}{\frac{28-72}{28}} = \frac{-117}{28} \cdot \frac{28}{-44} = \boxed{\frac{117}{44}}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25}$

$= \boxed{\frac{7}{25}}$

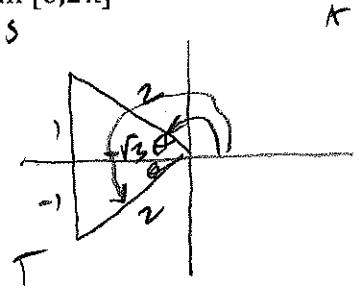
$\tan\frac{v}{2} = \frac{1-\cos v}{\sin v}$

$= \frac{1 - \frac{-7}{25}}{\frac{24}{25}} = \frac{25+7}{25} \cdot \frac{25}{24} = \frac{32}{24} = \boxed{\frac{4}{3}}$

Do the following problems WITHOUT a calculator!

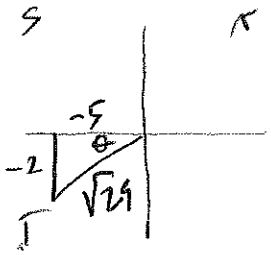
10) Find the exact value of angle θ in $[0, 2\pi]$

$\cos\theta = -\frac{\sqrt{3}}{2}$



$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

11) Evaluate $\sin\theta$, if $\tan\theta = \frac{2}{5}$ and $\cos\theta < 0$.



$\sin\theta = -\frac{2}{\sqrt{29}} = \boxed{-\frac{2\sqrt{29}}{29}}$

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12) Find the five remaining trig functions of θ satisfying the condition:

$\sin\theta = \frac{-3}{7}$, $\cos\theta > 0$

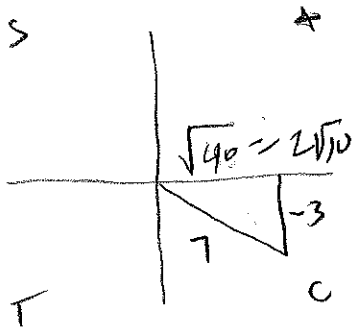
$\cos\theta = \frac{2\sqrt{10}}{7}$

$\tan\theta = \frac{-3}{2\sqrt{10}} = \frac{-3\sqrt{10}}{20}$

$\csc\theta = -\frac{7}{3}$

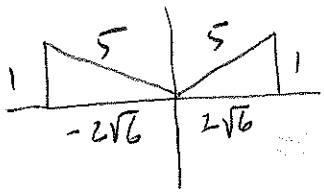
$\sec\theta = \frac{7}{2\sqrt{10}} = \frac{7\sqrt{10}}{20}$

$\cot\theta = -\frac{2\sqrt{10}}{3}$



13) Use the given function value and trig identity to find the indicated trig functions:

$\csc\theta = 5 \Rightarrow \sin\theta = \frac{1}{5}$



a) $\sin\theta = \frac{1}{5}$ b) $\cos\theta = \pm\frac{2\sqrt{6}}{5}$ c) $\sec\theta = \pm\frac{5\sqrt{6}}{12}$ d) $\tan\theta = \pm\frac{\sqrt{6}}{12}$

14) Show why the following are true

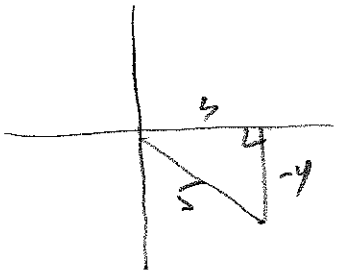
A) $\sin\theta\csc\theta = 1$

$\sin\theta\left(\frac{1}{\sin\theta}\right) = 1$ ✓

B) $\cos\theta\csc\theta = \cot\theta$

$\cos\theta\left(\frac{1}{\sin\theta}\right) = \cot\theta$ ✓

15) Find the EXACT values of the six trig functions of the angle θ (in standard position) whose terminal side passes through the point $(3,-4)$



$\sin\theta = -4/5$ $\cos\theta = 3/5$

$\tan\theta = -4/3$ $\cot\theta = -3/4$

$\sec\theta = 5/3$ $\csc\theta = -5/4$

16) Solve the following Trig Equations to find the ANGLE(S) in domain $[0,2\pi)$:

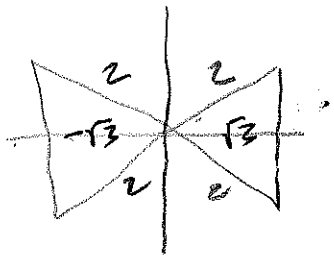
a) $3\sec^2 x - 4 = 0$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$X = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$



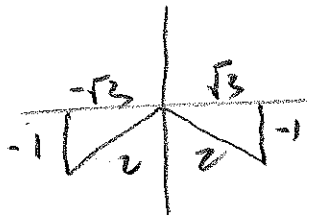
b) $\sin 2x + \cos x = 0$

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

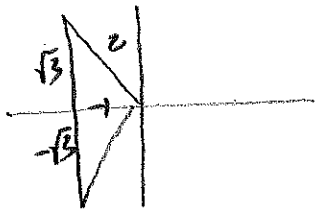


c) $2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

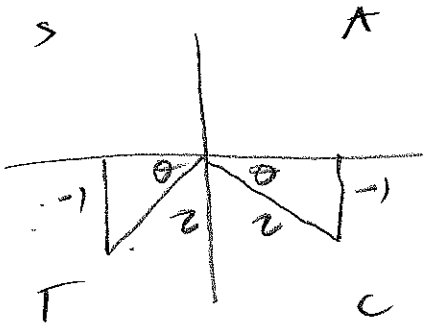
$x = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad x = 0, \pi$



17) Evaluate the following (it may help to draw a picture – look for special right triangles).Find two angles as your answers.

a) $\sin\theta = -\frac{1}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

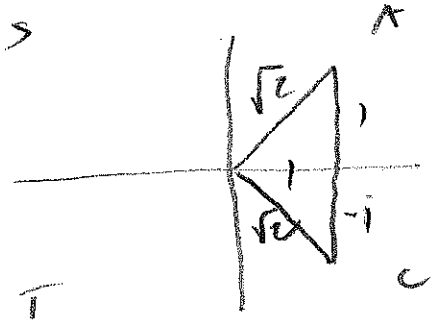


degrees: 210°

330°

b) $\cos\theta = \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}, \frac{7\pi}{4}$



degrees: 45°

315°

18) PROVE the trigonometric identities- be sure to only use ONE side!

a) $\sin^2\beta-\cos^2\beta=2\sin^2\beta-1$

$\sin^2-(1-\sin^2) = \sin^2-1+\sin^2$

$= 2\sin^2\beta-1$ ✓

b) $\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}=\csc\alpha\sec\alpha$

$\frac{\sin^2+\cos^2}{\sin\cos} = \frac{1}{\sin\cos} = \csc\alpha\sec\alpha$ ✓

c) $\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin x}\cdot\tan x=1$

$\frac{\cos x}{\sin x}\cdot\frac{\sin x}{\cos x} = 1$ ✓

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19) Find the amplitude and period of the following equations:

a) $y = -6\sin\frac{x}{2}$

amp = 6

period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

b) $y = \frac{1}{7}\cos 4\left(x + \frac{\pi}{2}\right)$

amp = $1/7$

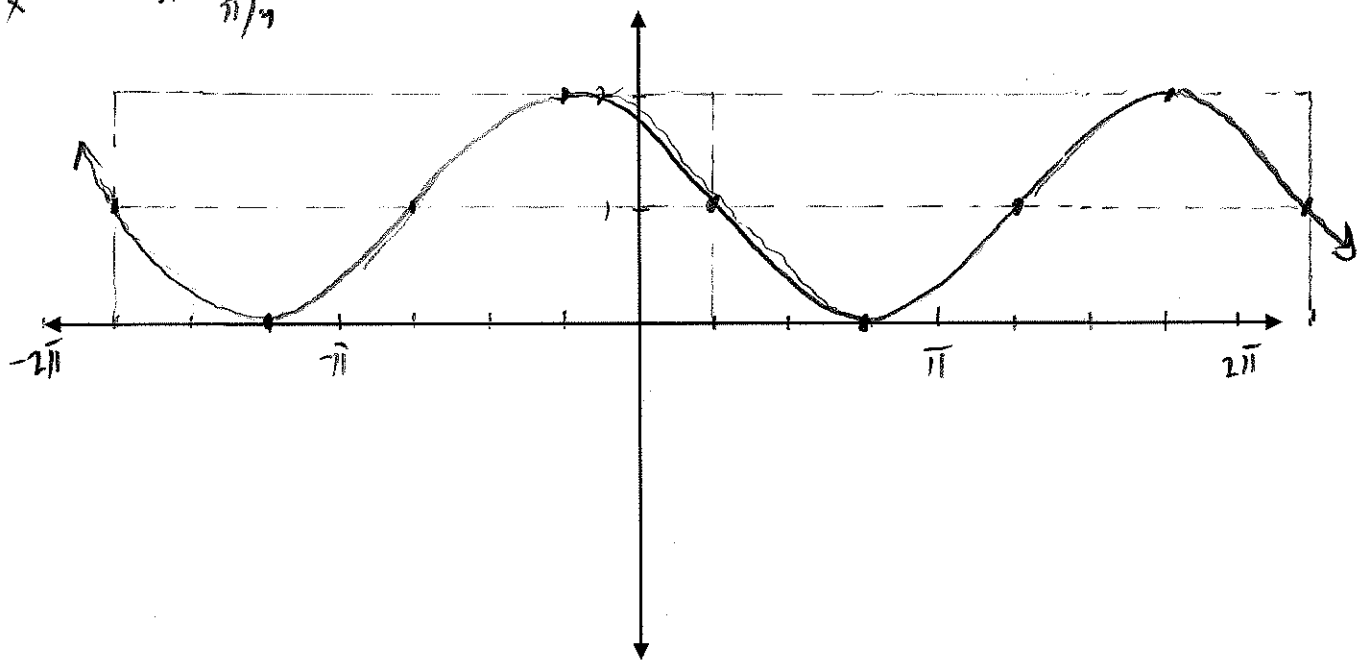
period = $\frac{2\pi}{4} = \frac{\pi}{2}$

20) Graph the following functions from -2π to 2π.

a) $y = -\sin\left(x - \frac{\pi}{4}\right) + 1$ up 1

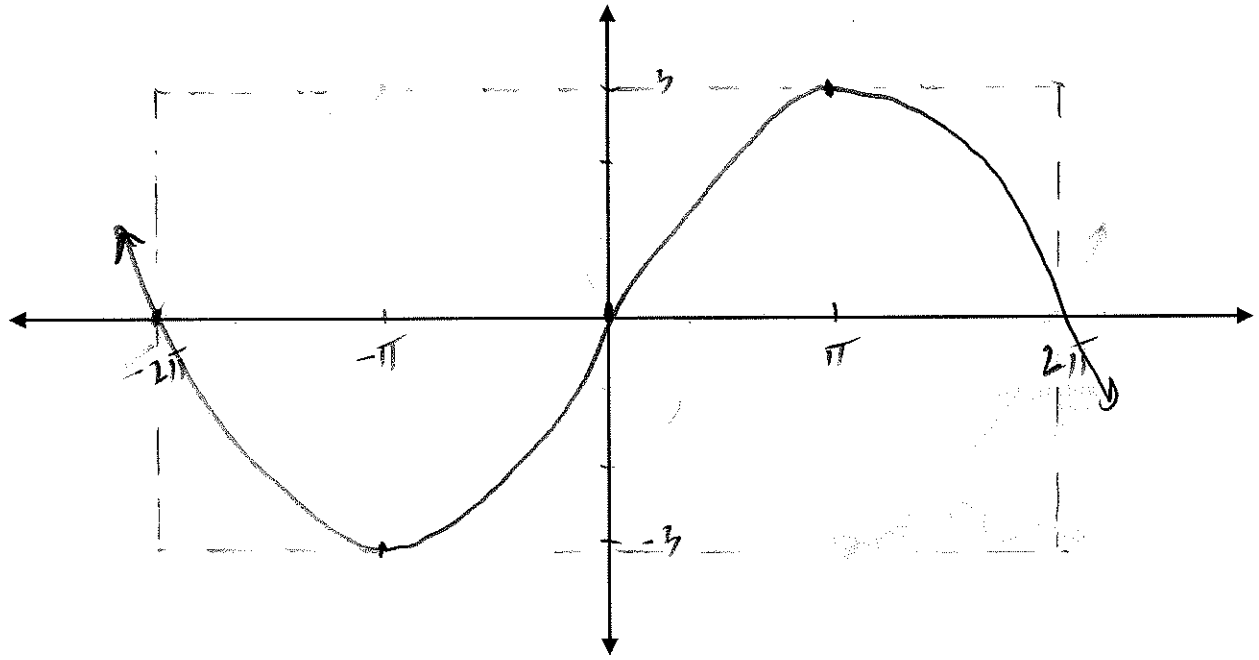
reflect over x

shift right $\pi/4$



amp $\frac{2\pi}{1/2} = 4\pi$

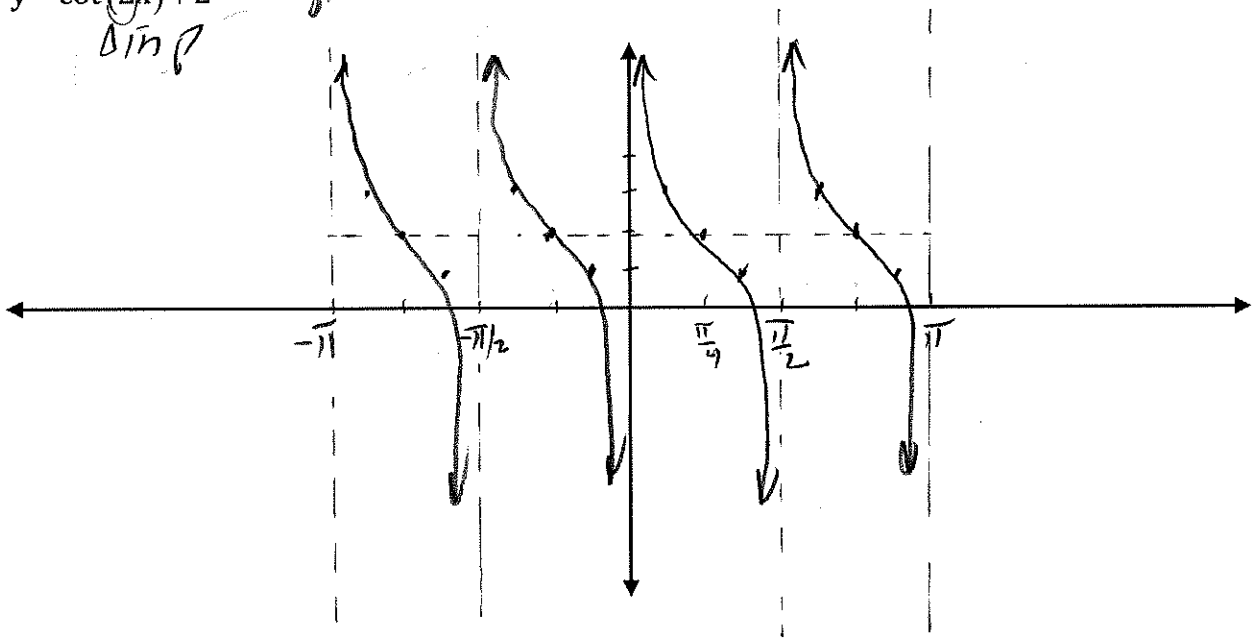
b) $y = 3\sin\left(\frac{1}{2}x\right)$



21) Graph the following functions from $-\pi$ to π.

a) $y = \cot(2x) + 2$ — up 2

Diff P

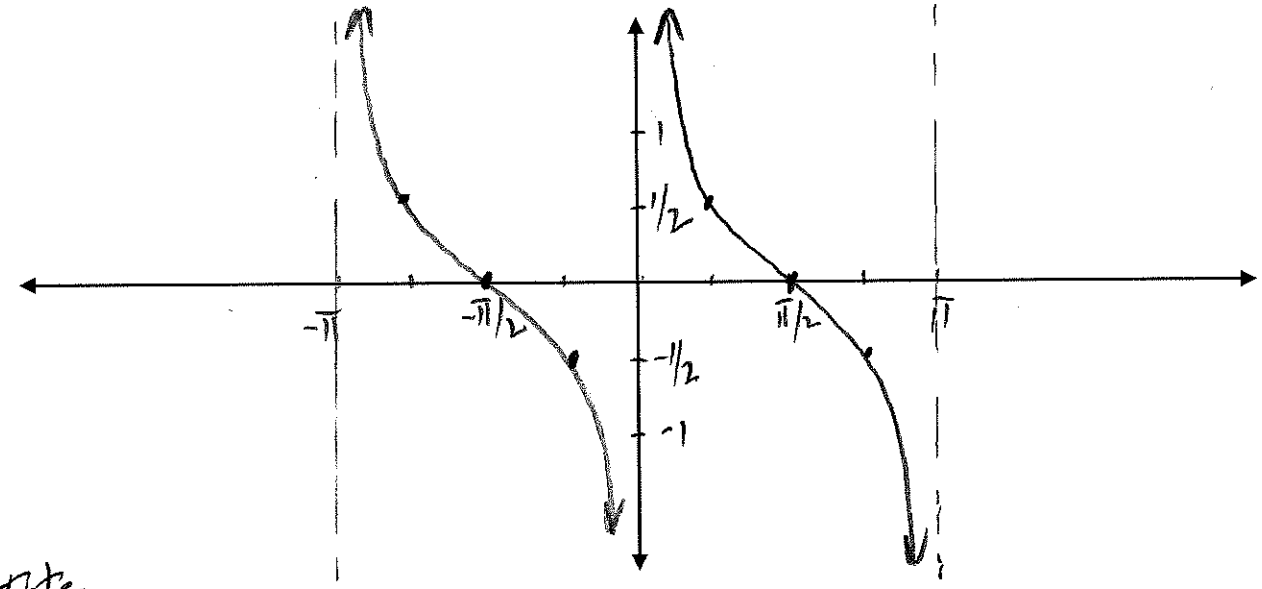


Asymptotes:

$2x = 0$ $\quad 2x = \pi$

$x = 0$ $\quad x = \pi/2$

b) $y = -\frac{1}{2}\tan\left(x - \frac{\pi}{2}\right)$ amp, shift right $\pi/2$, repeat π



Asymptotes

$x - \frac{\pi}{2} = \frac{\pi}{2}$ $\quad x - \frac{\pi}{2} = -\frac{\pi}{2}$

$x = \pi$ $\quad x = 0$

The following is review from chapter 1- this will not be a big part of the midyear exam, but there will be a few questions (multiple choice only), so it is worth it to review this stuff again!

22) Find the standard form of the equation of the circle described:
center at (-7, 5) & r = 9

$(x+7)^2 + (y-5)^2 = 81$

23) In what Quadrant is (x,y) located if $y > 0$ and $x < -4$?

II | I

III | IV

II

24) $f(x)=7x^2-1$

a) What type of symmetry does $f(x)$ have?

$f(-x)=7(-x)^2-1=7x^2-1=f(x)$ → y-axis

b) Is $f(x)$ even, odd, or neither?

even

25) $f(x)=\frac{5}{\sqrt{x-4}}$

a) What is the domain of $f(x)$?

$\{x \mid x>4\}$ or $(4,\infty)$

b) Find $f^{-1}(x)$

$x=\frac{5}{\sqrt{y-4}} \Rightarrow \sqrt{y-4}=\frac{5}{x}$

$y-4=\frac{25}{x^2}$

$y=\frac{25}{x^2}+4$

c) What is the domain of $f^{-1}(x)$?

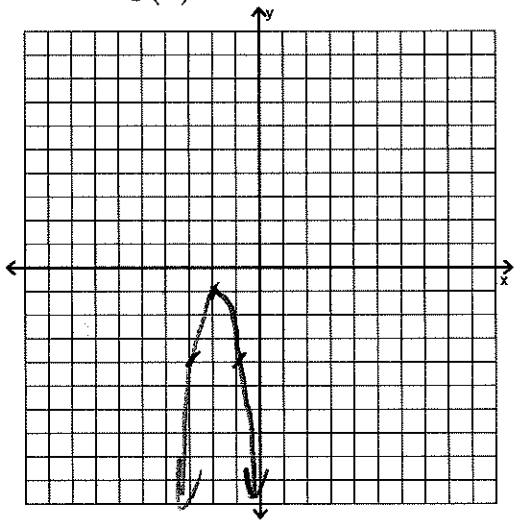
26) If $f(x)=-3(x+2)^2-1$

a) What is $f(4)$?

$f(4)=-3(4+2)^2-1=-3(36)-1=-108-1=-109$

b) Describe the transformation compared to the parent function $g(x)=x^2$ and sketch a graph.

reflect over x
vert stretch by fact. of 3
shift left 2
shift down 1



c) What are the domain and range of $f(x)$?

D: $(-\infty,\infty)$

R: $(-\infty,-1]$